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Class : 6th Sem 'A' Sec

USN : 1KS18ME011

Sub : FEM

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Ist INTENALS (I.A)

1a) **Simple Elements**

→ The elements which approximating polynomial consists of constant and linear terms ~~and~~ is called as simple elements.

→ The simple elements consists of corner nodes

→ They have triangular element with 3 nodes

The polynomial are given by

1 D → $u(x) = \alpha_1 + \alpha_2 x$

2 D → $u(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y$

3 D → $u(x,y,z) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 z$

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Complex elements

→ The elements which approximating polynomial consists of Quadratic, cubic & higher order terms in addition to the constant & linear terms are known as complex elements.

→ They may have the same shapes as the simple elements, but consisting of additional boundary nodes & sometimes internal nodes.

→ A triangular element with 3 corner nodes & 3 midside nodes satisfies the requirement

→ It ~~they~~ must have 6 nodes

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Quadratic model:

1 D = $u(x) = \alpha_1 + \alpha_2 x + \alpha_3 x^2$

2 D = $u(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2$

3 D = $u(x,y,z) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 z + \alpha_5 x^2 + \alpha_6 xy + \alpha_7 y^2 + \alpha_8 yz + \alpha_9 z^2 + \alpha_{10} zx$

Cubic model:

For 1d : $u(x) = \alpha_1 + \alpha_2 x + \alpha_3 x^2 + \alpha_4 x^3$

2D = $u(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^3 + \alpha_8 x^2 y + \alpha_9 xy^2 + \alpha_{10} y^3$

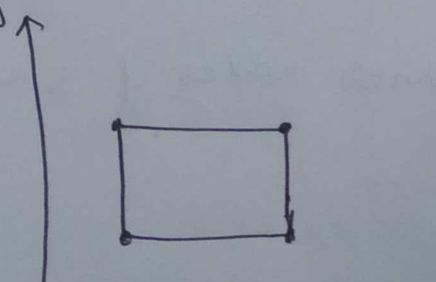
3D = $u(x,y,z) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 z + \alpha_5 x^2 + \alpha_6 xy + \alpha_7 y^2 + \alpha_8 xyz + \alpha_9 z^2 + \alpha_{10} zx + \alpha_{11} x^3 + \alpha_{12} x^2 y + \alpha_{13} xy^2 + \alpha_{14} y^3 + \alpha_{15} y^2 z + \alpha_{17} z^3 + \alpha_{18} z^2 x + \alpha_{19} zx^2 + \alpha_{20} xyz$

Multiplex elements

-> The elements whose approximating polynomial are higher order terms.

-> ~~For~~ In these elements boundaries are parallel to the coordinate axis to achieve inter element continuity

-> Rectangular element is an example of a multiplex elements in two dimension.



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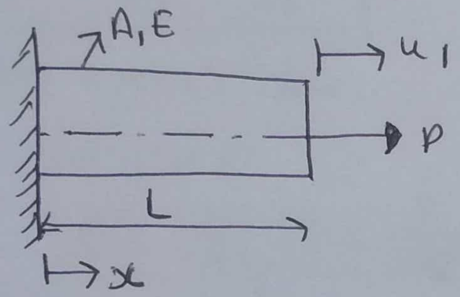
Interpolation polynomial

It is a approach for finding & representing displacement variation with the help of a mathematical model of a polynomial functions where the solution is approximate

Ex: $u(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$

1b) Soln



Step 1 : formulate the PE functional

$$\boxed{\Pi = SE + WP}$$

$$SE = \frac{AE}{2}\int_0^L \left(\frac{\partial u}{\partial x}\right)^2 dx$$

$$WP = -Pu_1$$

$$\boxed{\therefore \Pi = \frac{AE}{2}\int_0^L \left(\frac{\partial u}{\partial x}\right)^2 dx - Pu_1} \rightarrow (1)$$

wkt, $SE = \frac{1}{2}\int \sigma \epsilon \, dv$

from MOM, $\sigma = E\epsilon$

FEM, $\epsilon = \frac{\partial u}{\partial x}$

$dv = A\,dx$

$$\therefore SE = \frac{1}{2}\int_0^L (E\epsilon)\epsilon * A\,dx$$

$$= \frac{AE}{2}\int_0^L \epsilon^2 dx$$

$$\boxed{SE = \frac{AE}{2}\int_0^L \left(\frac{\partial u}{\partial x}\right)^2 dx}$$

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Step 2: Assume a polynomial displacement function

$$\boxed{u = a_0 + a_1 x} \rightarrow (2)$$

@ $x = 0$, $u = 0$ $\rightarrow$ $\boxed{a_0 = 0}$

Substitute $a_0 = 0$ in eq (2), we get

$$\boxed{u = a_1 x} \rightarrow (3)$$

diff wrt x

$$\boxed{\frac{\partial u}{\partial x} = a_1} \rightarrow (4)$$

@ $x = L$, $u = u_1$ $\rightarrow$ from eq (3)

$$\boxed{u_1 = a_1 2} \rightarrow (5)$$

Step 3: Substitute the displacement function into PE functional

$$\Pi = \frac{AE}{2} \int_0^L a_1^2 \, dx - P u_1$$

$$\Pi = \frac{AE}{2} a_1 [x]_0^L - P a_1 L$$

$$\Pi = \frac{AE}{2} a_1^2 L - P a_1 L \rightarrow (6)$$

Step 4 Minimise the PE functional

$$\frac{\partial \Pi}{\partial a_1} = \frac{AE}{\cancel{2}} \cancel{2} a_1 L - PL = 0$$

$$AE a_1 \cancel{L} = P \cancel{L}$$

$$\boxed{a_1 = \frac{P}{AE}}$$

Step 5 : determination of displacement strain & stress

Substituting the value at a_1 into eq (3)

$$\boxed{u = \left(\frac{P}{AE}\right) x} \rightarrow (7) = \text{Displacement}$$

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To find end deflection & stress

wkt, $x = L$ at end

$\therefore u = \left(\frac{P}{AE}\right)L$

from eq 7 we have,

$u = \left(\frac{P}{AE}\right)x$

$$\boxed{\epsilon = \frac{\partial u}{\partial x} = \frac{P}{AE}}$$ Strain

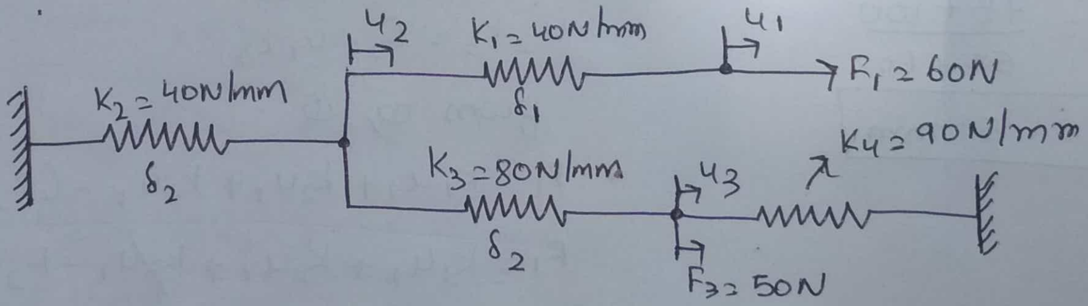
from hooke's law,

$\sigma = \epsilon E$

$\sigma = \left(\frac{\partial u}{\partial x}\right)E = \frac{P}{A\cancel{E}}\cancel{E}$

$$\boxed{\sigma = \frac{P}{A}}$$

1c)



Soln

wkt, $\boxed{\Pi = SE + WP}$ $\rightarrow$ (1)

$\rightarrow$ (SE) spring $= \frac{1}{2}K\delta^2$

$\rightarrow$ (SE) System $= \frac{1}{2}K_1\delta_1^2 + \frac{1}{2}K_2\delta_2^2 + \frac{1}{2}K_3\delta_3^2 + \frac{1}{2}K_4\delta_4^2$

Let u_1, u_2 & u_3 be the nodal displacement,

$\therefore \delta_1 = u_1 - u_2$, $\delta_2 = u_2$, $\delta_3 = u_3 - u_2$, $\delta_4 = -u_3$

$\therefore$ from eq (1)

(SE) system $= \frac{1}{2}K_1(u_1 - u_2)^2 + \frac{1}{2}K_2u_2^2 + \frac{1}{2}K_3(u_3 - u_2)^2 + \frac{1}{2}K_4u_3^2 \rightarrow$ (2)

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$(wp)_{system} = -F_1 u_1 - F_3 u_3 \to 3$

Subs eq ②, ③ in ① we get

$$\Pi = \frac{1}{2} k_1 (u_1 - u_2) + \frac{1}{2} k_2 u_2^2 + \frac{1}{2} k_3 (u_3 - u_1)^2 + \frac{1}{2} k_4 u_3^2 - F_1 u_1 - F_3 u_2$$

For PE to be minimum, $\frac{\partial \Pi}{\partial u_1} = 0$, $\frac{\partial \Pi}{\partial u_2} = 0$ $\frac{\partial \Pi}{\partial u_3} = 0$

$$\frac{\delta \Pi}{\delta u_1} = \frac{1}{2}\left[2u_1(k_1 + k_2 + k_3) - 2u_2 k_3\right] - F_1 = 0$$

$$\boxed{F_1 = k_1 u_1 + k_2 u_2 + k_3 u_1 - k_3 u_2} \to ①$$

$$\frac{\delta \Pi}{\delta u_2} = \frac{1}{2}\left[2u_2 k_3 - 2u_1 k_3\right] - F_2 = 0$$

$$\boxed{F_2 = u_2 k_3 - u_1 k_3} \to ②$$

solving eq ① & ② we get

$$u_1 = \frac{F_1 + F_2}{k_1 + k_2}$$

$$u_1 = \frac{75 + 100}{50 + 60}$$

$$\boxed{u_1 = 1.59 \text{ mm}}$$

From eq ②,

$u_2 k_3 = F_2 + u_1 k_3$

∴ from eq ①

$F_1 = k_1 u_1 + k_2 u_1 + k_3 u_1 - (F_2 + u_1 k_3)$

$F_1 = k_1 u_1 + k_2 u_1 + \cancel{k_3} u_1 - F_2 - u_1 \cancel{k_3}$

$(F_1 + F_2) = (k_1 + k_2) u_1$

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from eq ②, $\boxed{u_2 = 3.019 \text{ mm}}$

∴ Modal displacement, $\boxed{\begin{array}{l} u_1 = 1.59 \text{ mm} \\ u_2 = 3.019 \text{ mm} \end{array}}$

$$\frac{\partial \Pi}{\delta u_1} = \frac{1}{2} k_1 (2u_1 - 2u_2) - F_1 = 0$$

$$\boxed{F_1 = k_1 u_1 - u_2 k_1} \to ④$$

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$$\frac{\delta\pi}{\delta u_2} = \frac{1}{2}k_1(2u_2-2u_1) + k_2u_2 + \frac{1}{2}k_3(2u_2-2u_3) = 0$$

$$\frac{1}{2}[k_1(2u_2-2u_1) + 2k_2u_2 + k_3(2u_2-2u_3)] = 0$$

$$\frac{2}{2}[k_1(u_2-u_1) + k_2u_2 + k_3(u_2-u_3)] = 0$$

$$\boxed{-k_1(u_1-u_2) + k_2u_2 - k_3(u_3-u_2) = 0} \rightarrow (5)$$

$$\frac{\delta\pi}{\delta u_3} = \frac{1}{2}k_3(2u_3-2u_2) + \frac{2k_4u_3}{2} - F_3 = 0$$

$$= \frac{1}{2}k_3(u_3-u_2) + k_4u_3 - F_3 = 0$$

$$k_3u_3 - k_3u_2 + k_4u_4 - F_3 = 0$$

$$\boxed{F_3 = -k_3u_2 + (k_3+k_4)u_3} \rightarrow (6)$$

from eq (4) (5) (6) we get

$$\left\{\begin{array}{l} F_1 = 60N, \quad F_2 = 0, \quad F_3 = 50N \\ k_1 = 40N/mm \quad k_3 = 80N/mm \\ k_2 = 40N/mm, \quad k_4 = 90N/mm \end{array}\right\}$$

$$\boxed{60 = 40u_1 - 40u_2} \rightarrow (a)$$

$$-40(u_1-u_2) + 40u_2 - 80(u_3-u_2) = 0$$

$$\boxed{-40u_1 + 160u_2 - 80u_3 = 0} \rightarrow (b)$$

$$50 = -80u_2 + (80+90)u_3$$

$$\boxed{50 = -80u_2 + 170u_3} \rightarrow (c)$$

$$\therefore \left.\begin{array}{l} 40u_1 - 40u_2 = 60 \\ -40u_1 + 160u_2 - 80u_3 = 0 \\ -80u_2 + 170u_3 = 50 \end{array}\right\}$$

on simplification

$u_1 = 2.514$ mm

$u_2 = 1.014$ mm

$u_3 = 0.7714$ mm //

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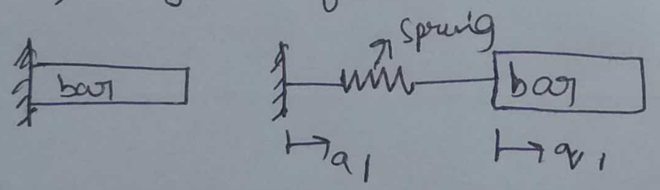
3a) Methods of handling Boundary Conditions

a) Elimination method
b) Penalty method

① Elimination Method

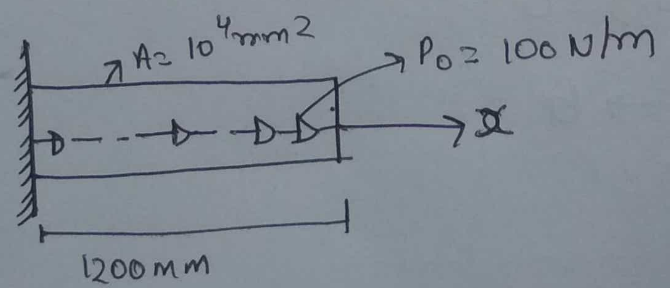
→ It is a method of handling boundary conditions used in FEM to solve the equilibrium equation after applying known boundary conditions. In this method, after apply Boundary condition, the row & the column corresponding to the known value of the field variable is eliminated or deleted.

(ii) Penalty approach : In the penalty method of handling boundary condition, fixed supports are modelled using a spring which is having large stiffness, C



$$\boxed{\therefore\ C = \max|k_{ij}| * 10^4}$$

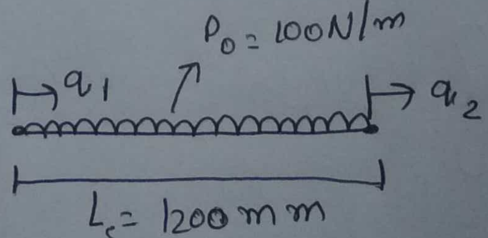
3b)



$P_0 = 100 N/m$
$= 100 \times 10^{-3} N/mm$
$= 0.1 N/mm$

Soln

① FE model



② Elemental Stiffness matrix

$$K = \frac{AE}{L_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{10^4 * 70\times10^3}{1200} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 10^5 \begin{bmatrix} 5.83 & -5.83 \\ 5.83 & 5.83 \end{bmatrix}$$

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⑨

Nodal displacement vector (Q)

$$(Q) = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ Q_2 \end{bmatrix} \rightarrow ②$$

④ Load vector due to UDL [F]

$$(F) = \frac{F_0 L}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{0.1 * 1200}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 60 \\ 60 \end{Bmatrix} \rightarrow ③$$

⑤ equilibrium eqn $[K]\{Q\} = \{F\}$

$$10^5\begin{bmatrix} 5.83 & -5.83 \\ -5.83 & 5.83 \end{bmatrix}\begin{Bmatrix} 0 \\ Q_2 \end{Bmatrix} = \begin{Bmatrix} 60 \\ 60 \end{Bmatrix}$$

From method of handling Boundary conditions (elimination method)

$$5.83 \times 10^5 Q_2 = 60$$

$$\boxed{\begin{aligned} Q_2 &= 102.91 * 10^{-6}\ mm \\ Q_1 &= 0 \end{aligned}}$$

$$\therefore \{Q\} = \begin{Bmatrix} 0 \\ 102.91 \times 10^{-6} \end{Bmatrix}$$

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⑥ Stress in each element

$$\sigma_1 = \sigma = \frac{E}{L}(Q_2 - Q_1) = \frac{70 \times 10^3}{1200}(102.91 \times 10^{-6} - 0)$$

$$\boxed{\sigma = 6 \times 10^{-3}\ N/mm^2}$$